

# Cornell Ranger: Energy-Optimal Control

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## Summary and Introduction

The Cornell Ranger is a powered walking biped that has recently walked over 9 kms non-stop on a single battery charge without human contact. The specific cost of transport for this walk was 0.55 (0.36 in motors and 0.19 in other electrical overhead). This is relatively small compared to most recent robots but is still thrice that of humans and also thrice that of the Collins walker. The goal of this work is to use computer simulations and numerical optimization to calculate and then implement energy-optimal walking gaits for this biped.

First, we present a relatively accurate mechanical computer model of Ranger. The previous fit of model parameters was presented last year at Dynamic Walking [1]. Next, we solve the energy-optimal control problem using parameter-optimization software and discover (surprise) that pre-emptive push-off is an energy saving strategy.

## The Model

The 2D model consists of two rigid legs connected to each other by a hip hinge at which there is an additional mass. The feet are mass-less and round. The robot is powered by three motors; one for the hip and one for each ankle. The ankle motors are connected to the feet through a linear, torsional spring ('series elastic actuation'). Also, there is zero-free-length spring pulling the legs to their parallel position. The motor model uses a linear fit to bench top measurements of torque, voltage, current and speed. The foot-to-ground collisions are assumed to be instantaneous and with no slip or bounce of feet.

## The energy-optimal control problem

We try to find that periodic motion which minimizes the specific cost of transport,

$$c_t = \frac{\text{energy cost}}{\text{weight} \times \text{distance}}$$
$$= \frac{\int_0^{t_{\text{step}}} (\sum \text{Motor Powers}) dt + \text{Overheads} \times t_{\text{step}}}{\text{mass} * g * \text{step length}}$$

Each motor power in the sum (above) is given by the product of voltage and current ( $P_i = V_i I_i$ ) and Overheads is fixed at 8 W. The optimization variables are the time to take one step ( $t_{\text{step}}$ ), all initial angles and velocities. The control functions to be optimized are motor voltages

which are parameterized as piecewise linear functions of time. We used an eighth order variable time stepping integrator for integrating the equations of motion and a sequential quadratic programming (SQP) [2] algorithm for the optimization of the resulting solutions.

To verify and develop our simulation and optimization, we carried out a few optimizations whose solutions we already knew. First, we locked the ankles, introduced a ramp and the optimization discovered passive dynamic walking. Second, we put all the mass in the hips, made the legs light and the optimization discovered the classical inverted pendulum walking with push-off before heel-strike (e.g., [3]).

To be clear, this optimization is for energy and not stability. A separate control problem is to stabilize motions to such an optimal gait. The means for this stability is the subject of another abstract here [4].

## Results

We found that the optimal gait corresponds to push off before heel-strike with passive leg swings and is consistent with earlier findings for humans (e.g., [3]). However, if the center of mass of the legs is moved sufficiently forward of the hip the control strategy changes from ankle to hip actuation. So far, the optimization suggests that we can reduce to a cost of transport to 0.15 (0.04 in motors and 0.11 in other electrical overhead).

## References

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